

Lattice QCD at High Temperature and the QGP

Frithjof Karsch, Brookhaven National Laboratory

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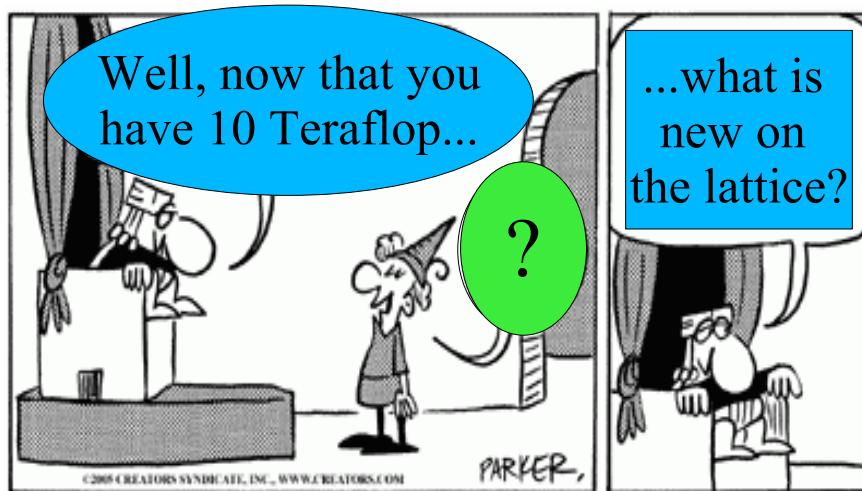


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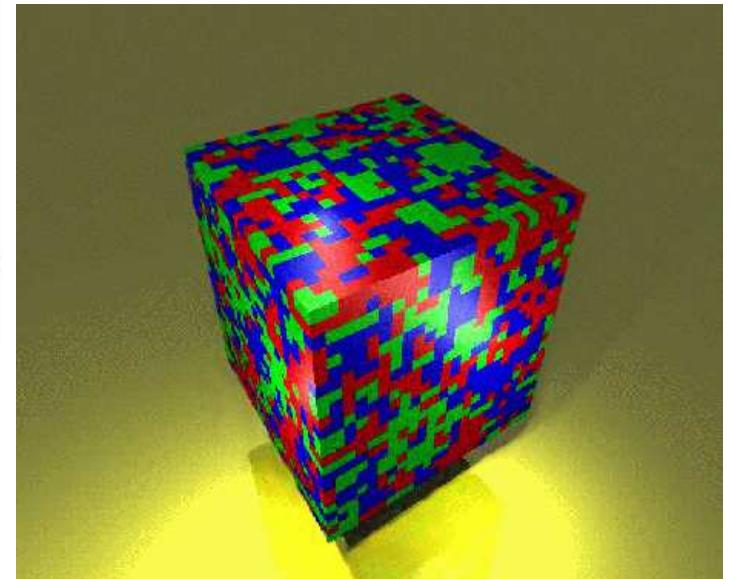


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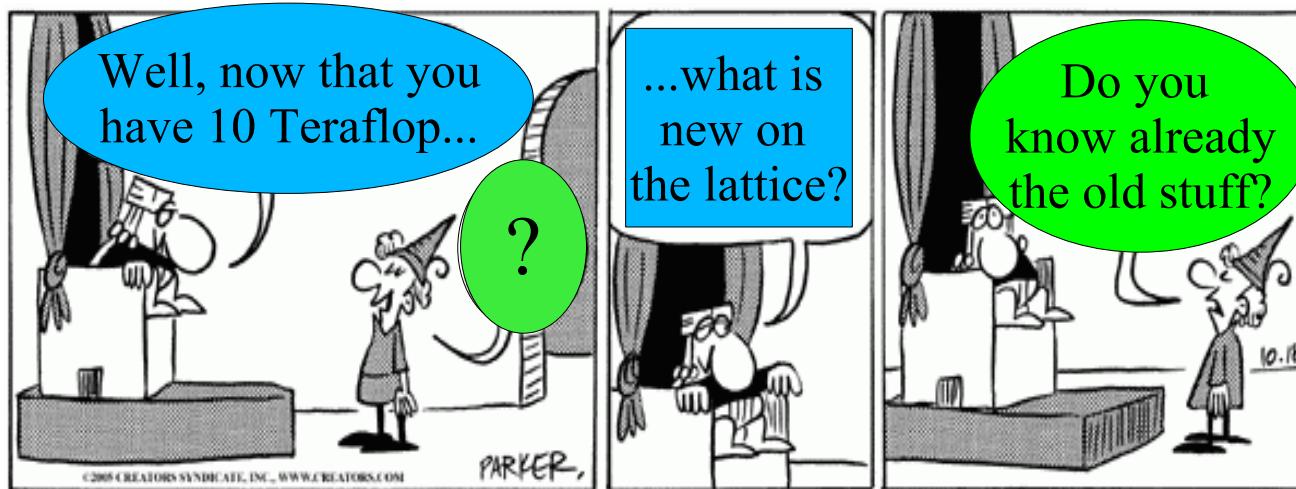


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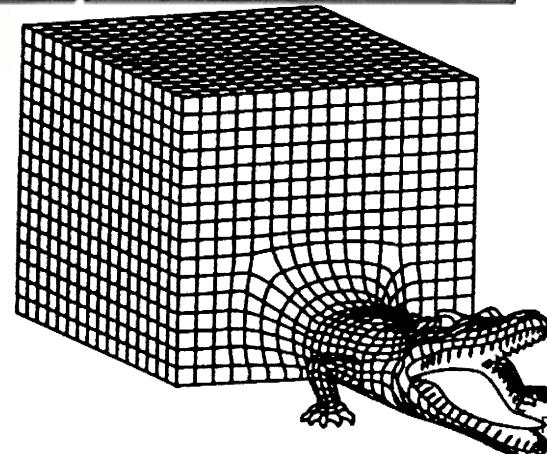


Lattice QCD at High Temperature and the QGP

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Many questions ...

The relatively easy (but already difficult) questions concern

- T_c , EoS, c_s , screening and the heavy quark potential,...

The more difficult questions concern

- details of the QCD phase diagram,
the order of the transition and the
location/existence of the chiral critical point

the really tough questions concern

- the structure of the high temperature phase,
properties of heavy quark bound states,
thermal dilepton rates and transport coefficients,
the fate of light quark bound states

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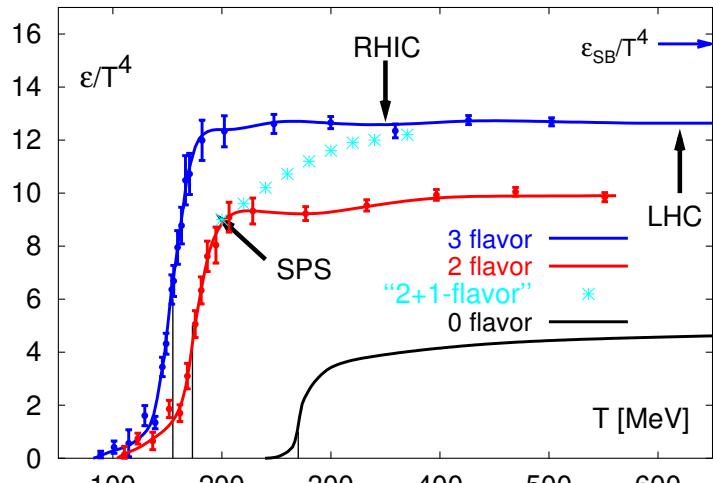
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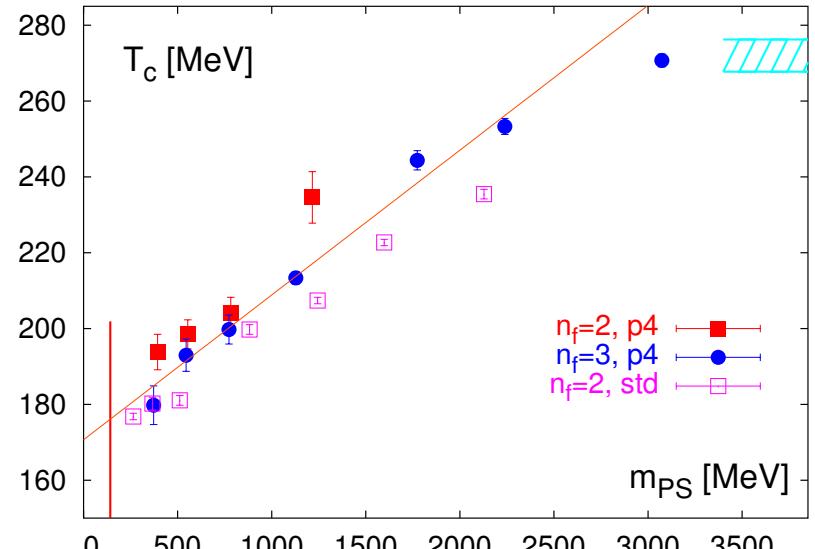
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Equation of State and T_c



QCD EoS



transition temperature

- ϵ/T^4 for $m_\pi \simeq 770$ MeV;
 $(m_\pi/m_\rho \simeq 0.7, TV^{1/3} = 4)$
 $\epsilon_c/T_c^4 = 6 \pm 2$ ⇒ $T_c = (173 \pm 8 \pm sys)$ MeV
 $(T_c$ for $m_\pi \gtrsim 300$ MeV)
 $\epsilon_c = (0.3 - 1.3) \text{GeV/fm}^3$
- improved staggered fermions but still on rather coarse lattices:
 $N_\tau = 4$, i.e. $a^{-1} \simeq 0.8$ GeV

FK, E. Laermann, A. Peikert, Nucl. Phys. B605 (2001) 579

recent results on T_c

attempt to extrapolate to chiral and continuum limit:

$$\Lambda T_c = c_0(m_\pi/m_\rho)^d + c_2(aT_c)^2$$

C. Bernard et al., Phys. Rev. D71 (2005) 034504

$\mathcal{O}(a^2)$ improved staggered fermions, (2+1)-flavor, $N_\tau = 6$, $TV^{1/3} = 2$
 $m_\pi/m_\rho \gtrsim 0.3$, $\Lambda \equiv r_1 = 0.317(7)\text{fm}$,

V.G. Bornyakov et al., hep-lat/0509122

$\mathcal{O}(a)$ improved Wilson fermions, 2-flavor, $N_\tau = 8, 10$, $TV^{1/3} \simeq 2$
 $m_\pi/m_\rho \gtrsim 0.4$, $\Lambda \equiv r_0 = 0.5\text{fm}$,

NOTE systematic errors: $\sim 5\%$ in scale setting, $\sim 5\%$ in extrapolation

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$$\Rightarrow \quad T_c = 169(12)(4) \quad (d = 2/\beta\delta = 1.08)$$
$$T_c = 174(11)(4) \quad (d = 2)$$

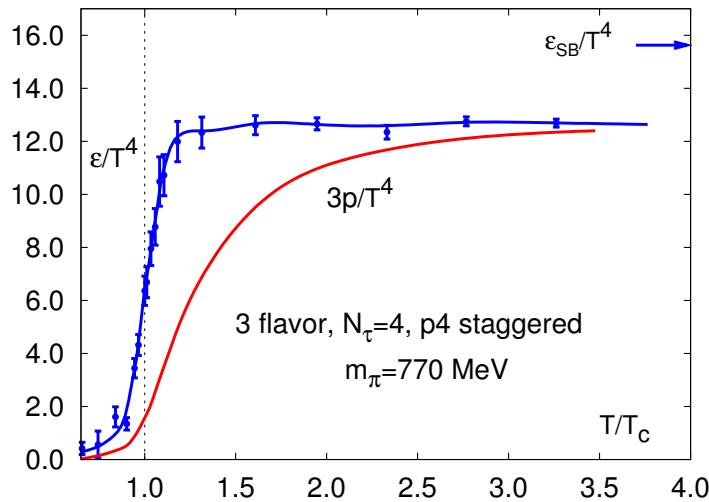
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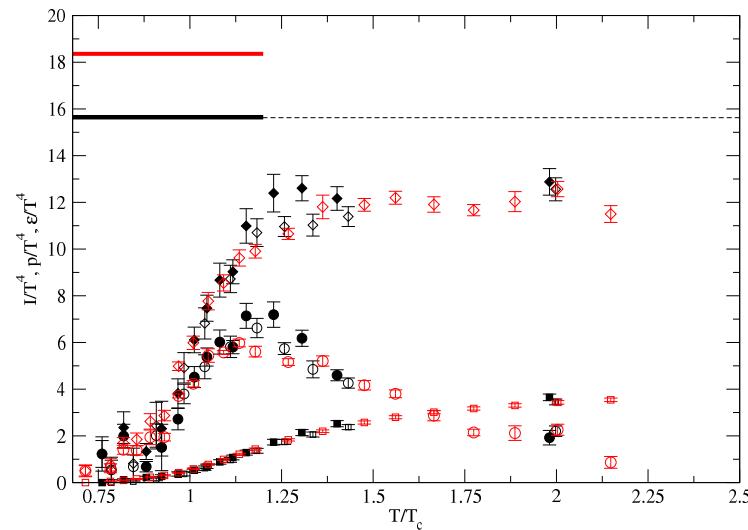
$$\Rightarrow \quad T_c = 166(3) \quad (d = 2/\beta\delta = 1.08)$$
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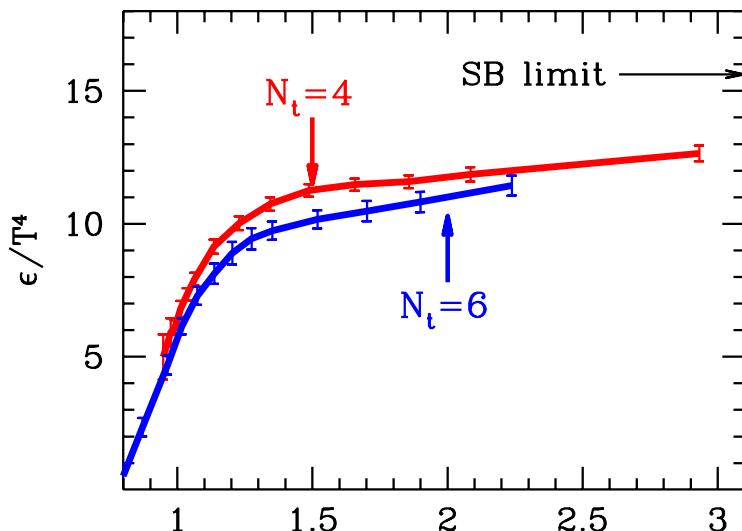
recent results on QCD EoS



old Bielefeld result, 2001
improved staggered (p4), $N_\tau = 4$
3-flavor, $m_\pi \simeq 770$ MeV

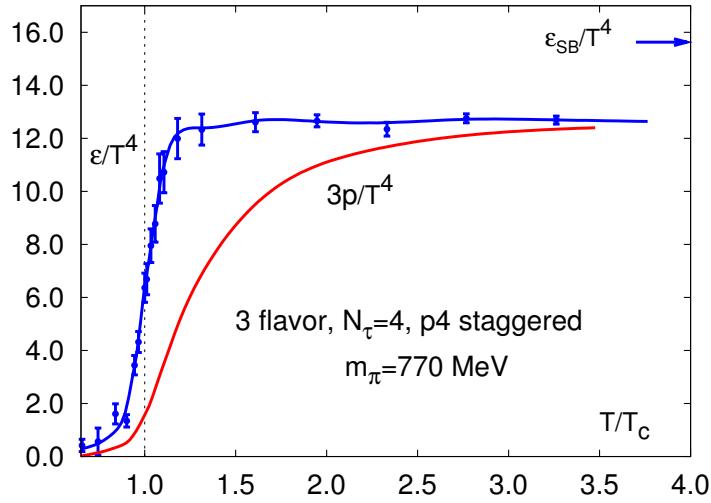


MILC-collaboration, hep-lat/0509053
 $\mathcal{O}(a^2)$ improved staggered, $N_\tau = 4, 6$
(2+1)-flavor, $m_\pi \gtrsim 250$ MeV

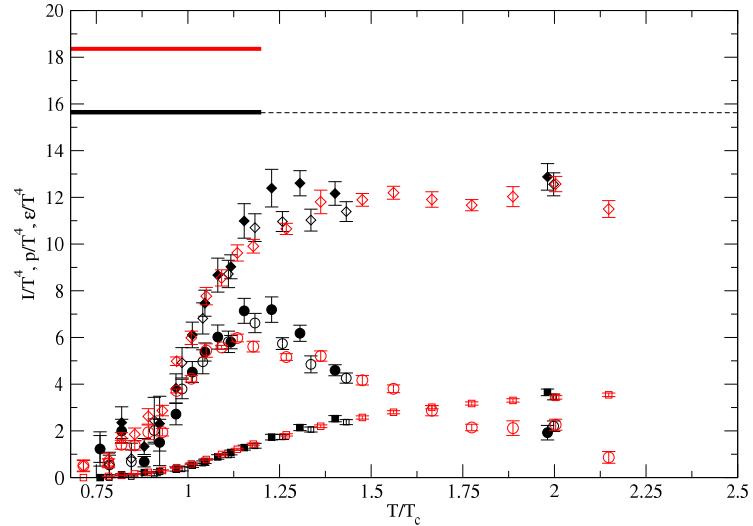


Y. Aoki et al., hep-lat/0510084
standard staggered, $N_\tau = 4, 6$
(2+1)-flavor, $m_\pi \rightarrow 140$ MeV (extrap.)
 ϵ/T^4 rescaled with $(\epsilon_{SB}/T^4)(N_\tau)$

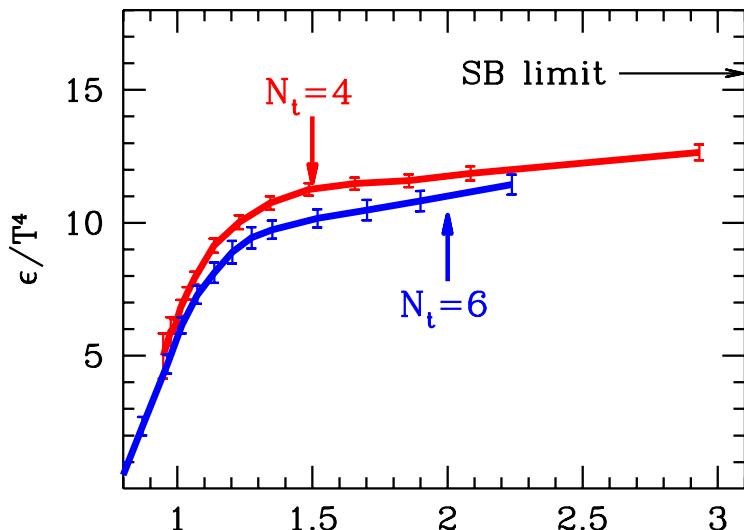
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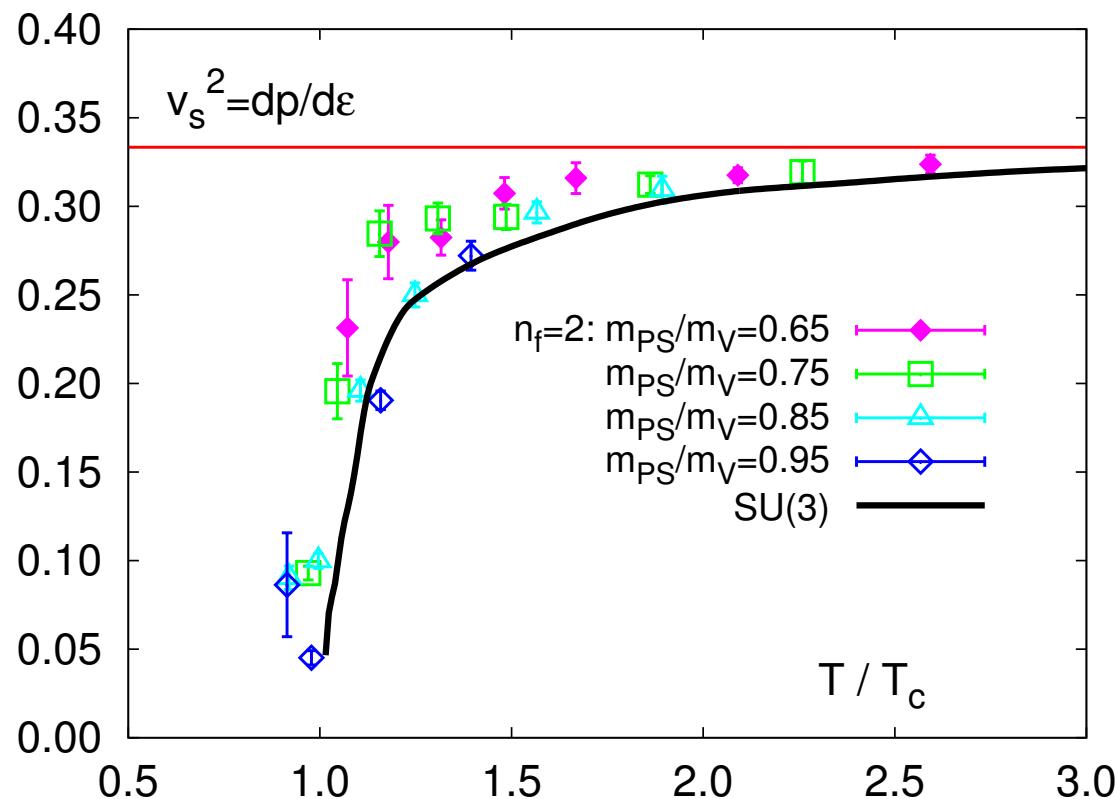


$\epsilon_c/T_c^4 \simeq 6$ insensitive to m_π and a^{-1}
HOWEVER: thermodynamic limit??
 $TV^{1/3} \simeq 2$

Y. Aoki et al., hep-lat/0510084
standard staggered, $N_\tau = 4, 6$
(2+1)-flavor, $m_\pi \rightarrow 140$ MeV (extrap.)
 ϵ/T^4 rescaled with $(\epsilon_{SB}/T^4)(N_\tau)$

Velocity of sound

- steep EoS:
 - rapid change of energy density; slow change of pressure
 - ⇒ reduced velocity of sound ⇒ more time for equilibration

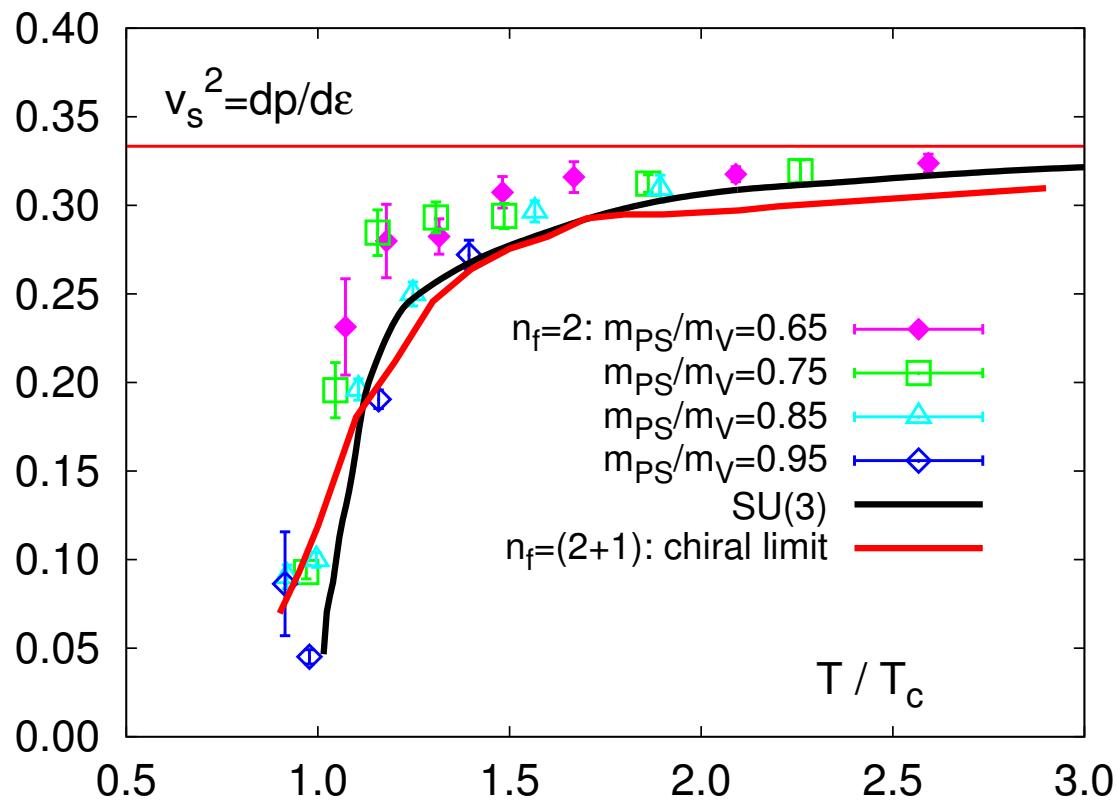


pure gauge theory:
G. Boyd et al.,
NP B469 1996

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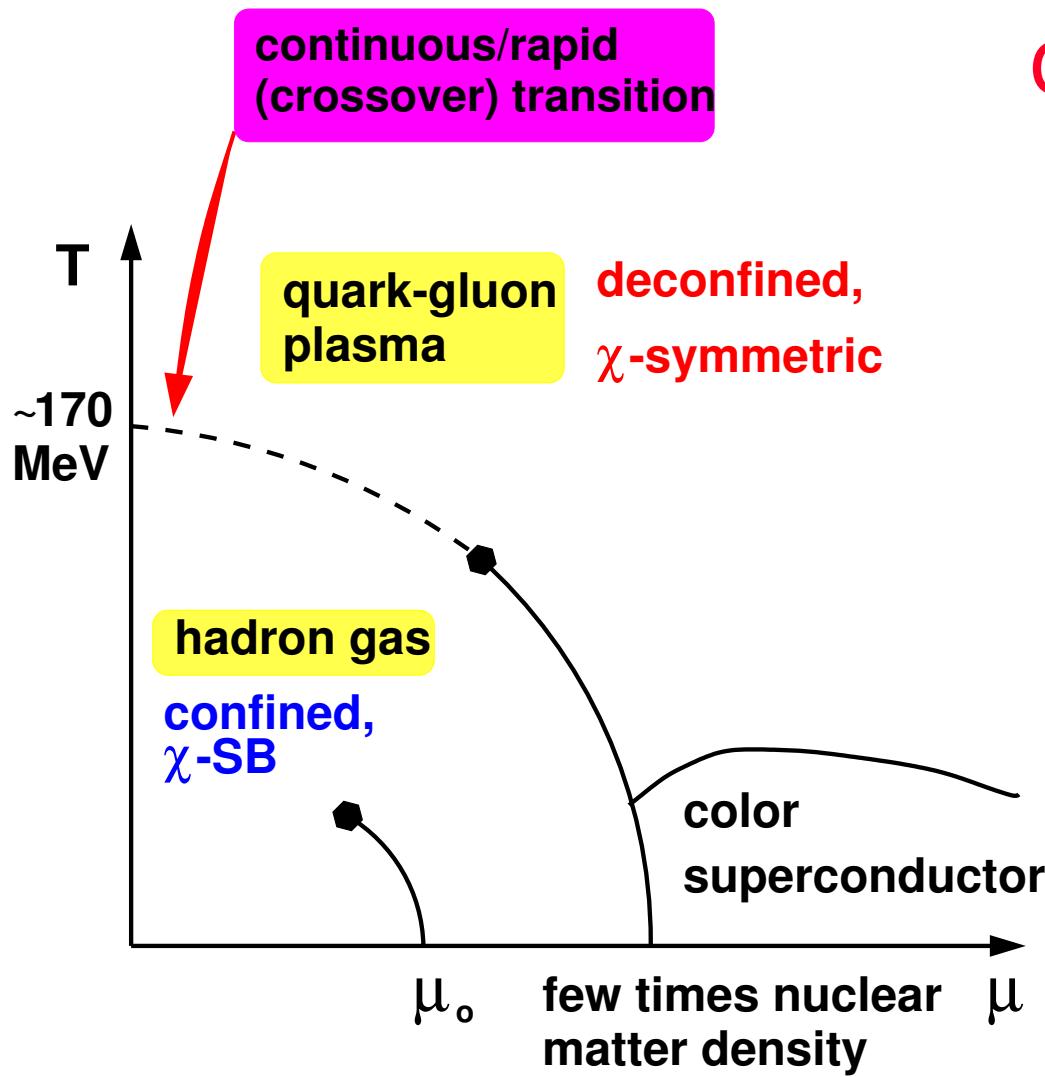


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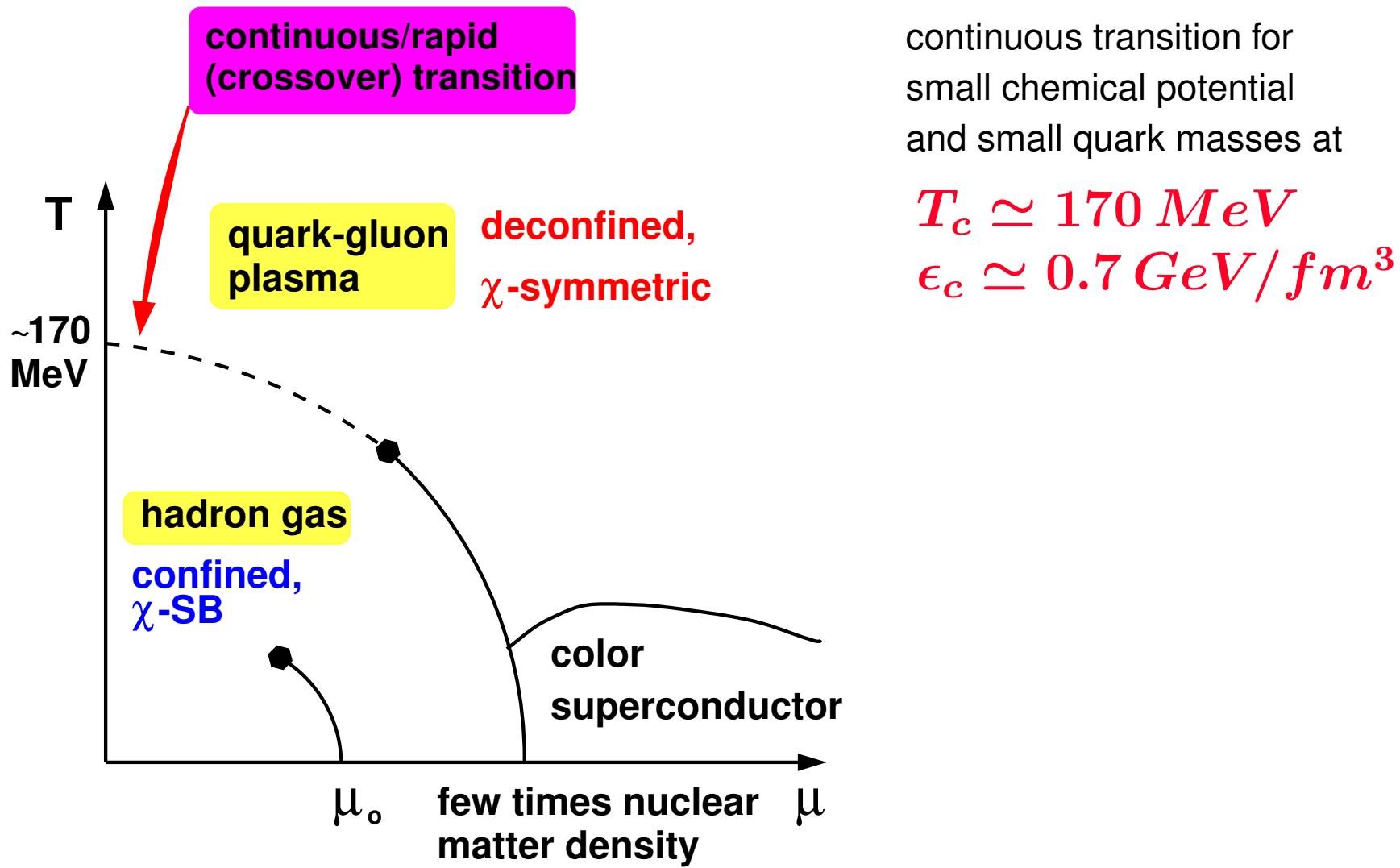
$n_f = (2 + 1)$:
Y. Aoki et al.,
hep-lat/0510084
($TV^{1/3} = 2$)

Critical behavior in hot and dense matter: QCD phase diagram



crossover vs.
phase transition

Critical behavior in hot and dense matter: QCD phase diagram

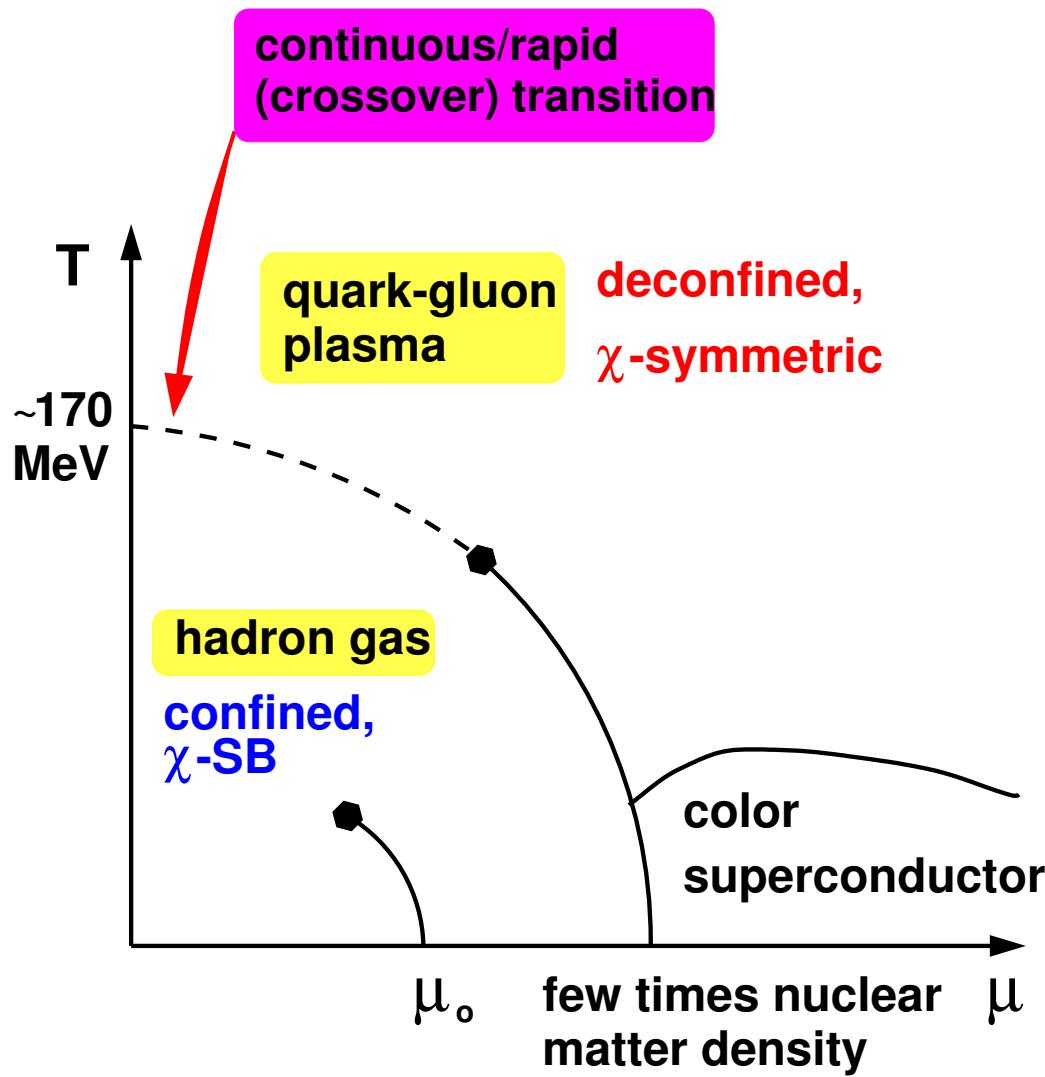


continuous transition for
small chemical potential
and small quark masses at

$$T_c \simeq 170 \text{ MeV}$$

$$\epsilon_c \simeq 0.7 \text{ GeV/fm}^3$$

Critical behavior in hot and dense matter: QCD phase diagram

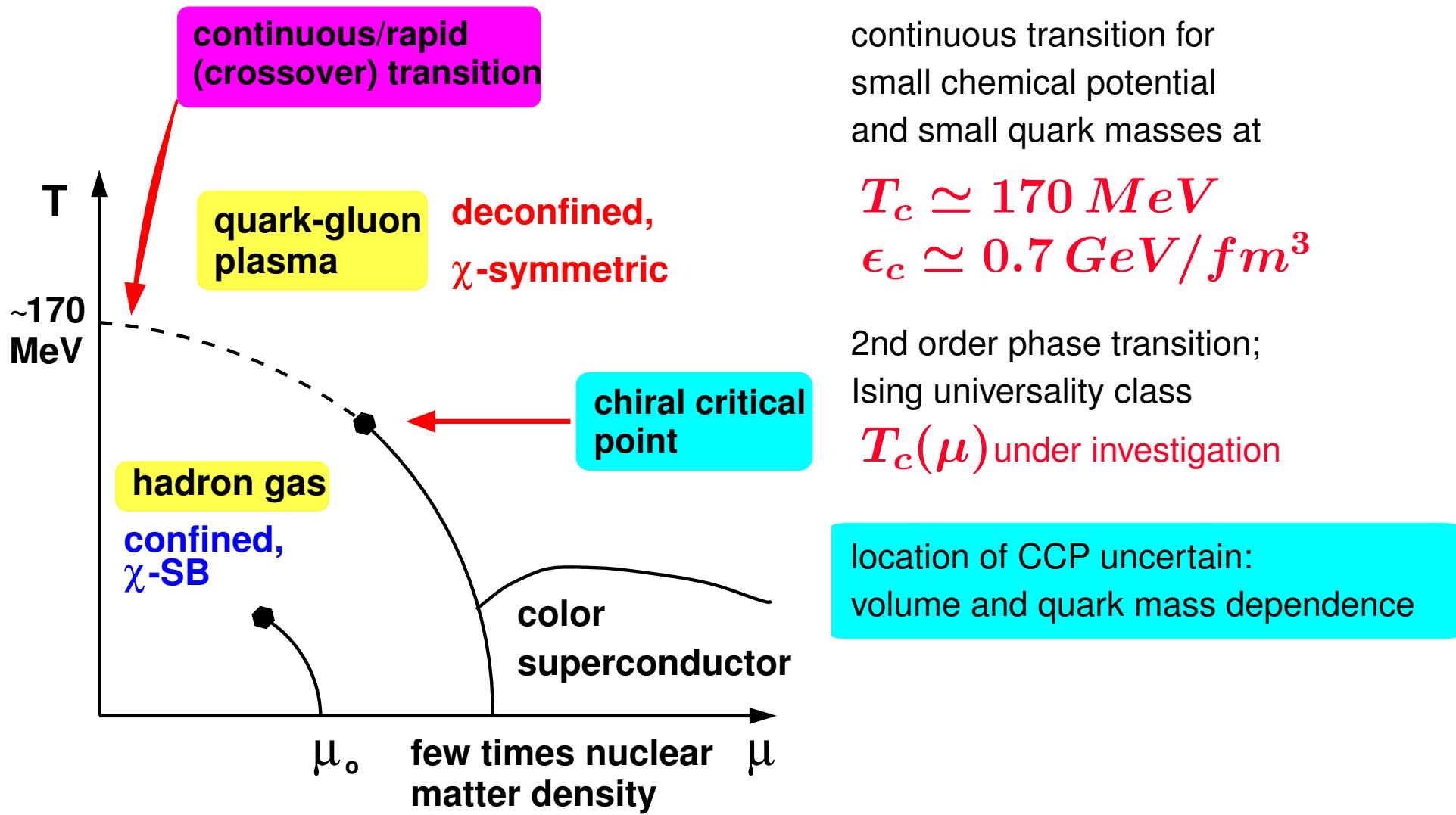


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recent doubts on order of transition
A. Di Giacomo et al., hep-lat/0503030
and PANIC 2005

Critical behavior in hot and dense matter: QCD phase diagram



Bulk thermodynamics with non-vanishing chemical potential

$$\begin{aligned} Z(\mathbf{V}, \mathbf{T}, \boldsymbol{\mu}) &= \int \mathcal{D}\mathcal{A} \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_E(\mathbf{V}, \mathbf{T}, \boldsymbol{\mu})} \\ &= \int \mathcal{D}\mathcal{A} [\det M(\boldsymbol{\mu})]^f e^{-S_G(\mathbf{V}, \mathbf{T})} \\ &\quad \uparrow \text{complex fermion determinant}; \end{aligned}$$

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\uparrow complex fermion determinant;

ways to circumvent this problem:

- **reweighting**: works well on small lattices; requires exact evaluation of $\det M$
Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- **Taylor expansion** around $\mu = 0$: works well for small μ ;
C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
R.V. Gavai, S. Gupta, Phys. Rev. D68 (2003) 034506
- **imaginary chemical potential**: works well for small μ ; requires analytic continuation
Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290
M. D'Elia and M.P. Lombardo, Phys. Rev. D64 (2003) 014505

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recent progress;

- **reweighting:** larger lattices; smaller quark mass;
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$\mu_B \sim 180$ MeV

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the dust still must settle

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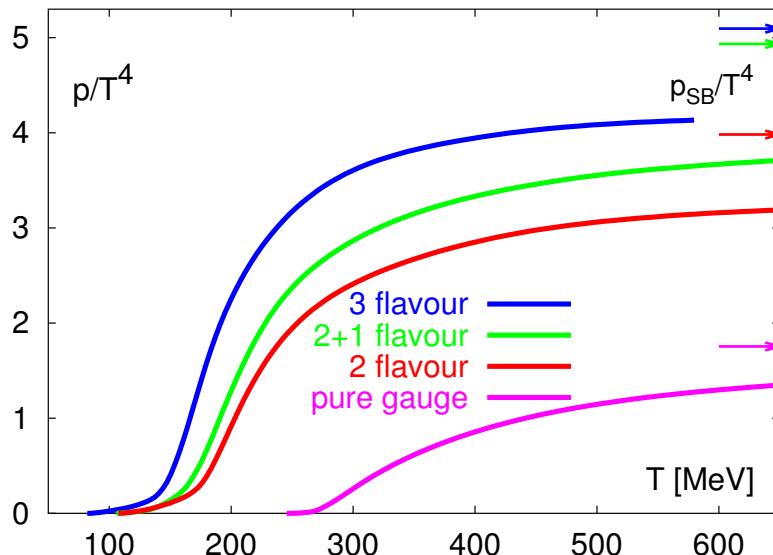
$$\begin{aligned} \frac{p}{T^4} &= \frac{1}{VT^3} \ln Z(\mathbf{V}, \mathbf{T}, \boldsymbol{\mu}) \\ &\equiv \sum_{n=0}^{\infty} c_n(T) \left(\frac{\boldsymbol{\mu}}{T} \right)^n \\ &= c_0 + c_2 \left(\frac{\boldsymbol{\mu}}{T} \right)^2 + c_4 \left(\frac{\boldsymbol{\mu}}{T} \right)^4 + \mathcal{O}((\boldsymbol{\mu}/T)^6) \end{aligned}$$

$$\boldsymbol{\mu} = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507

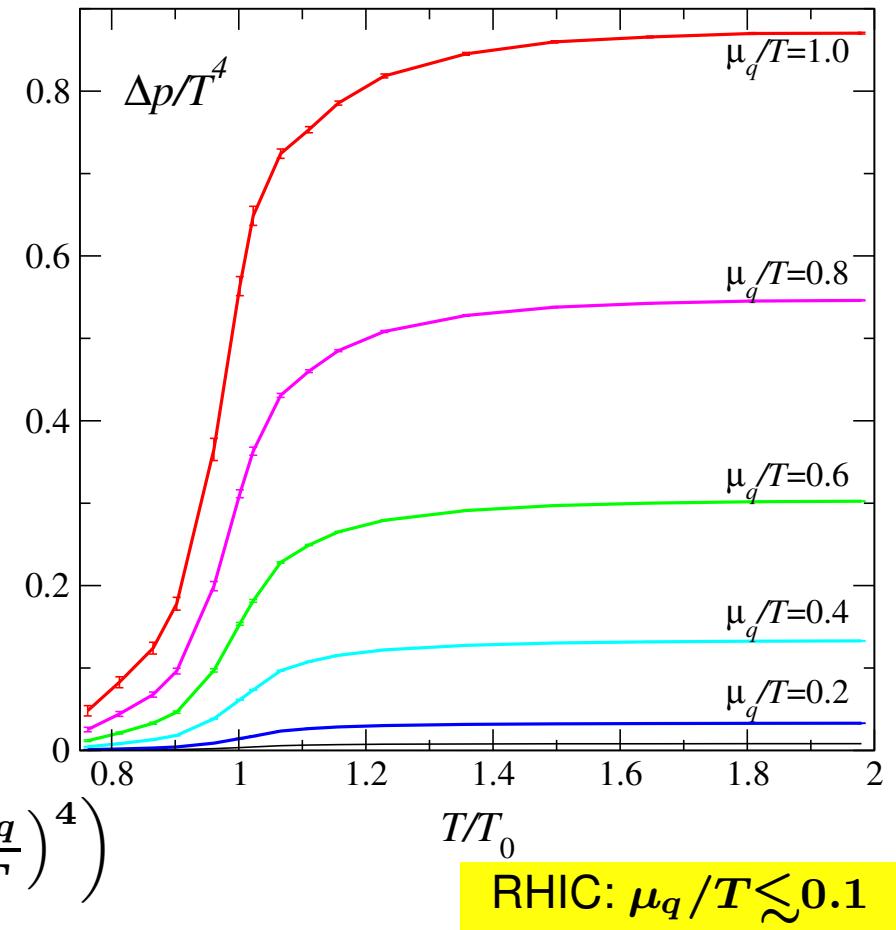
$\mu_q = 0$, $16^3 \times 4$ lattice
improved staggered fermions;
 $n_f = 2$, $m_\pi \simeq 770$ MeV



high-T, ideal gas limit

$$\left. \frac{p}{T^4} \right|_\infty = n_f \left(\frac{7\pi^2}{60} + \frac{1}{2} \left(\frac{\mu_q}{T} \right)^2 + \frac{1}{4\pi^2} \left(\frac{\mu_q}{T} \right)^4 \right)$$

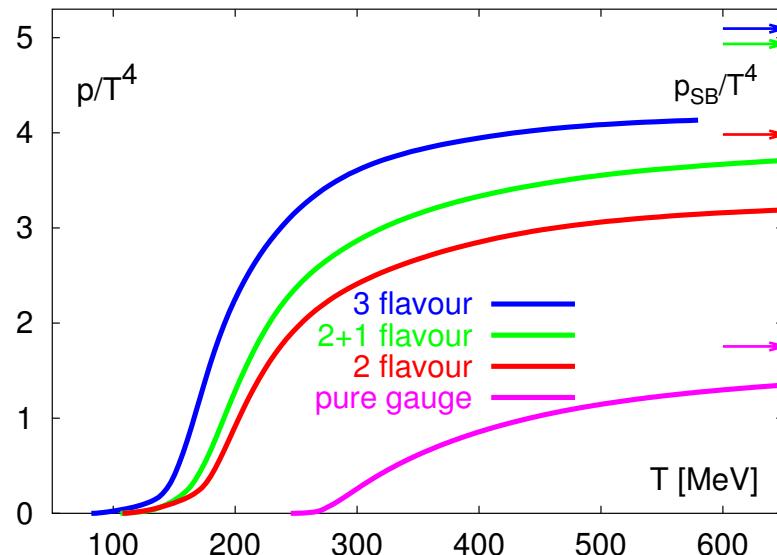
contribution from $\mu_q/T > 0$
Taylor expansion, $\mathcal{O}((\mu/T)^4)$



The pressure for $\mu_q/T > 0$

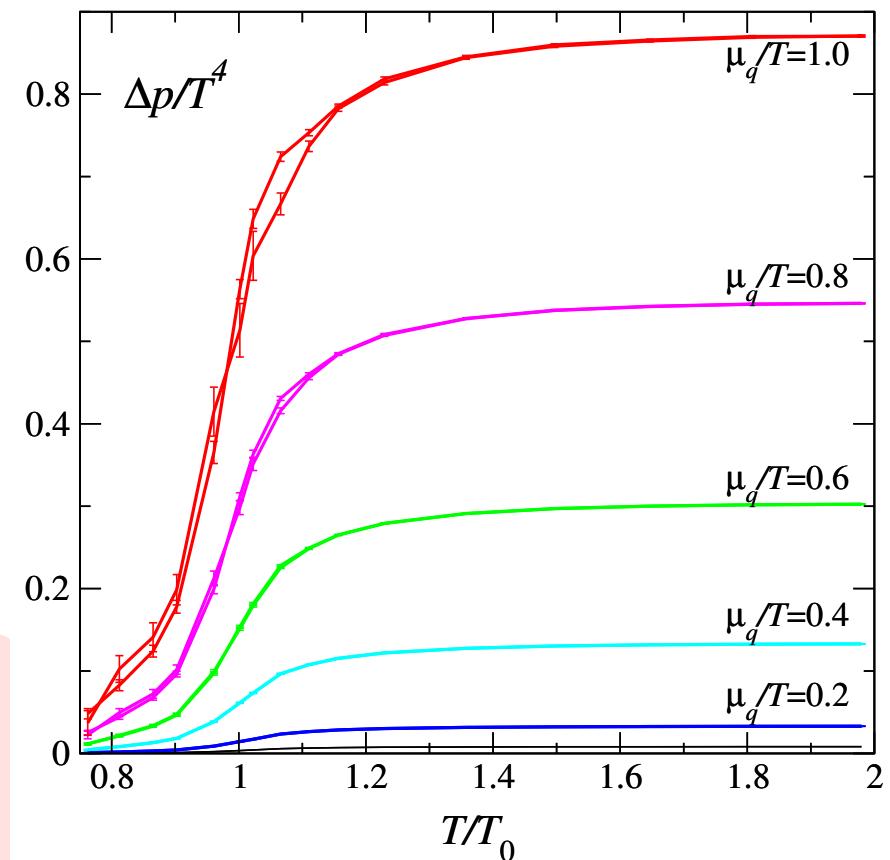
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pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
 $\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

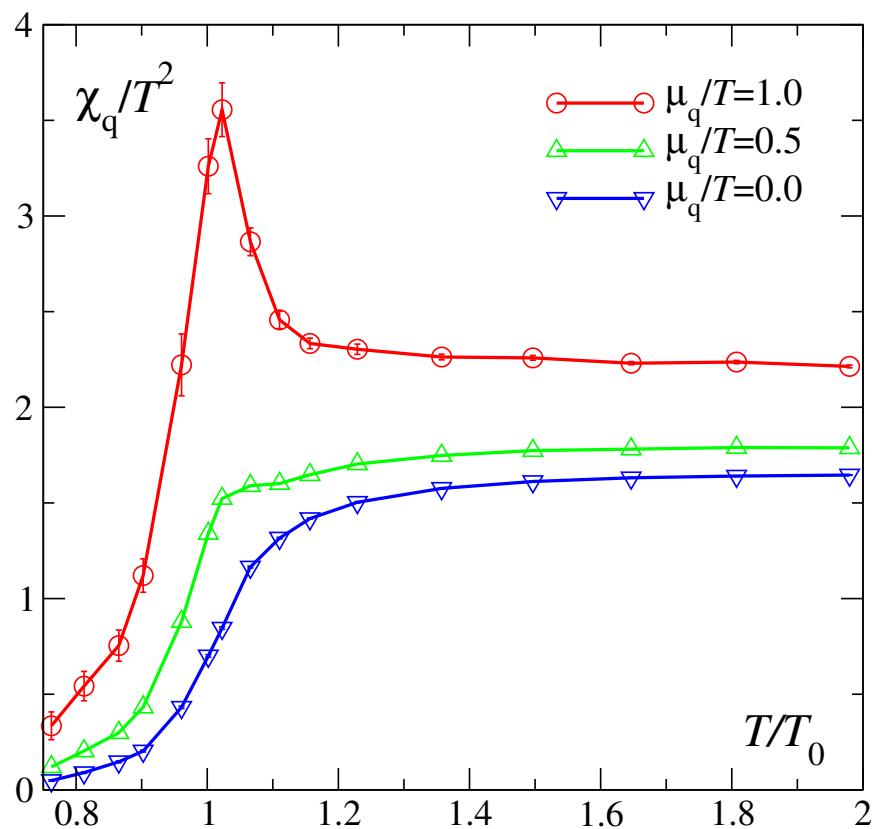
PRD71 (2005) 054508
contribution from $\mu_q/T > 0$
NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$



RHIC: $\mu_q/T \lesssim 0.1$

Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations:
up to $\mathcal{O}((\mu_q/T)^2)$



$$\frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu_q/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

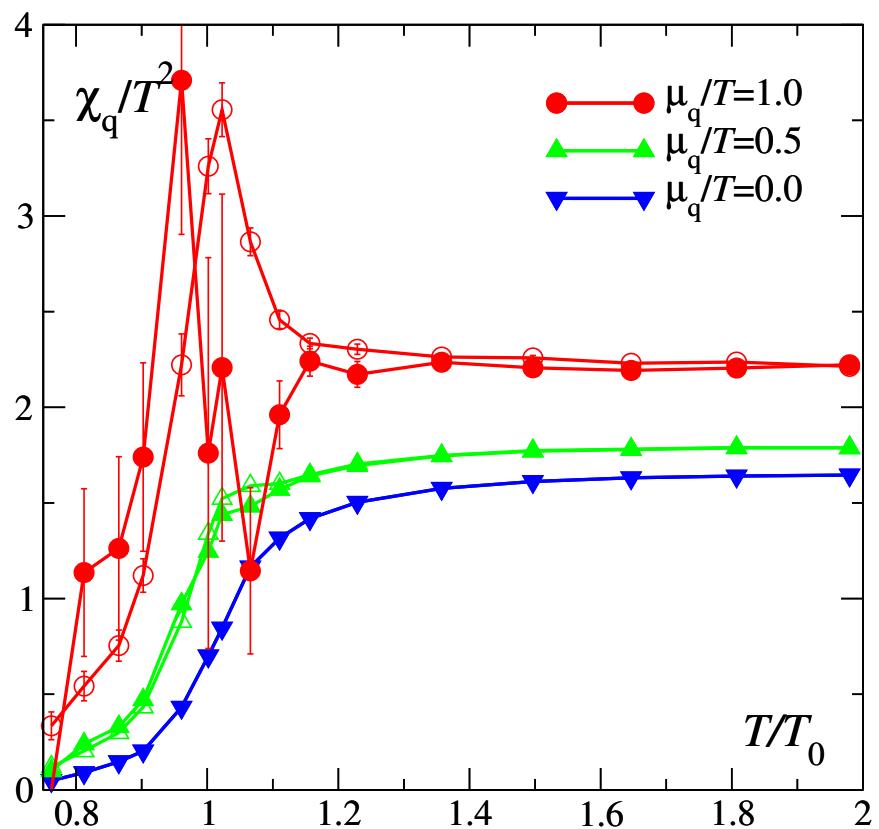
$$= \frac{1}{VT^3} \left(\langle N_q^2 \rangle - \langle N_q \rangle^2 \right)$$

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2} \left(\frac{\mu_q}{T} \right)^2$$

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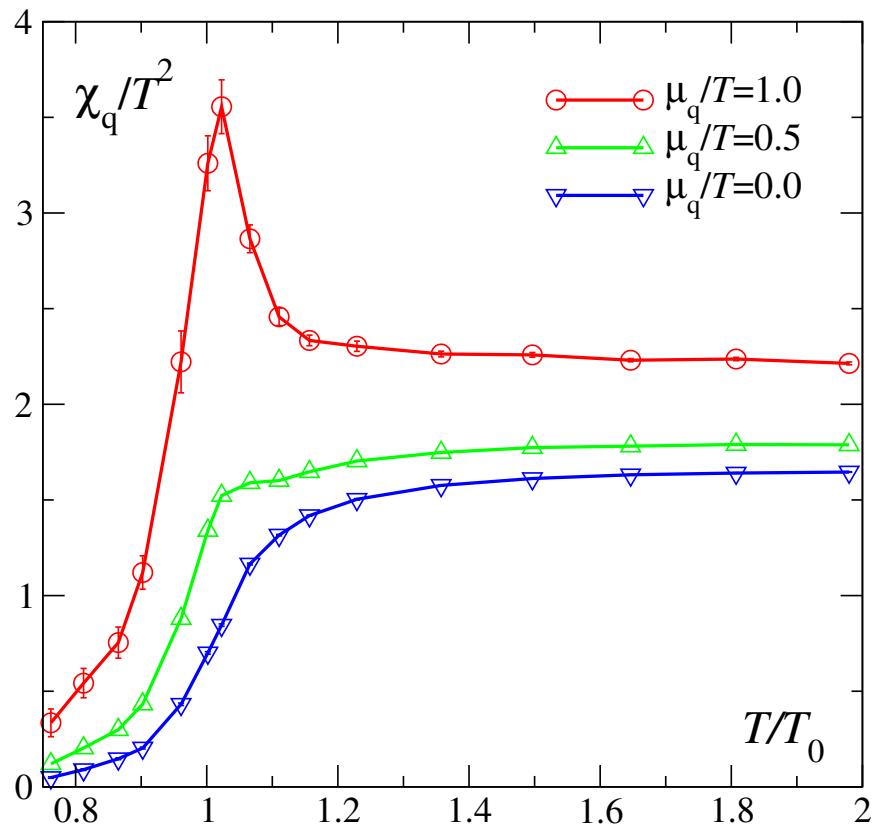
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quark number density fluctuations:

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$$= \frac{1}{VT^3} \left(\langle N_q^2 \rangle - \langle N_q \rangle^2 \right)$$

high-T, massless limit: polynomial in (μ_q/T)

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larger density fluctuations for $\mu_q > 0$;
coming closer to the chiral critical point?

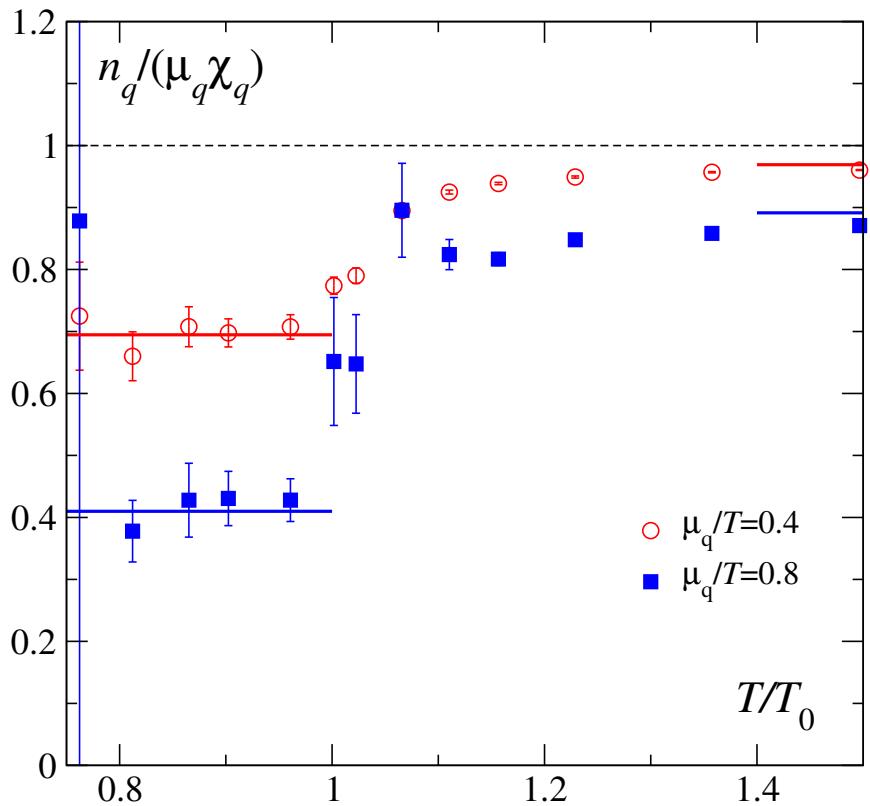
$$\left(\frac{\partial p}{\partial n_q} \right)_T = \frac{n_q}{\chi_q}$$

$\Rightarrow \chi_q$ will diverge on chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q} \right)_{T \text{ fixed}}$$



high-T, massless limit: polynomial in (μ_q/T)

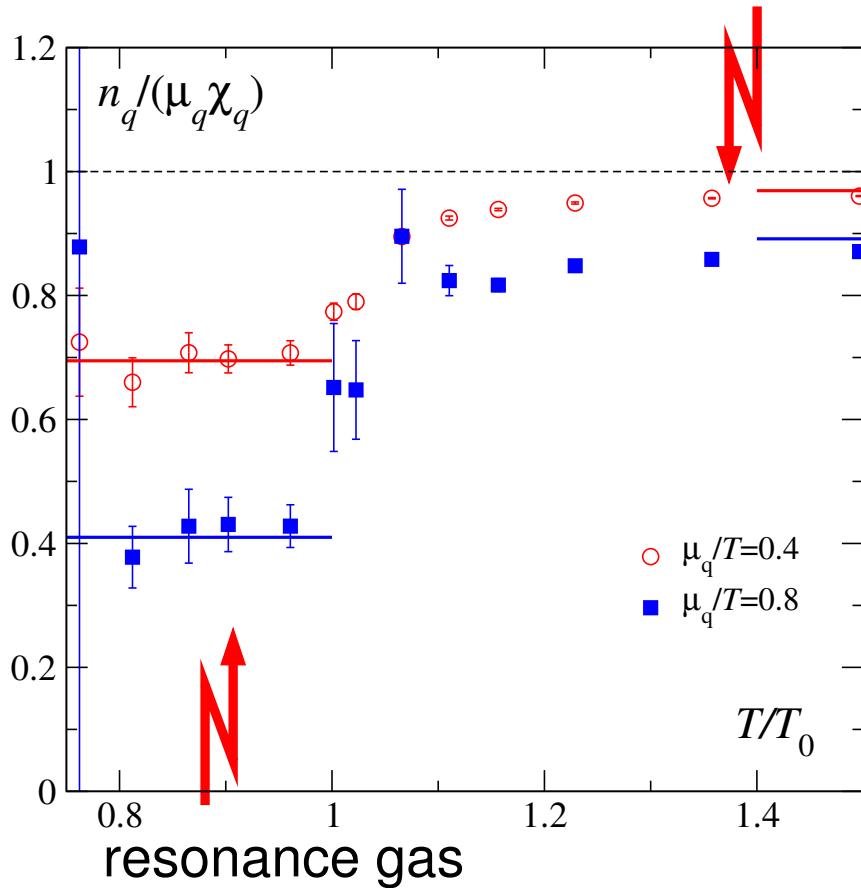
$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O} \left(\left(\frac{\mu_q}{T} \right)^3 \right)$$

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ideal $q\bar{q}$ gas



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large density fluctuations for $\mu_q > 0$, $T < T_c$

"saturated" by fluctuations in a
hadron resonance gas

expect: $\left(\frac{\partial p}{\partial n_q} \right)_T = \frac{n_q}{\chi_q} = 0$

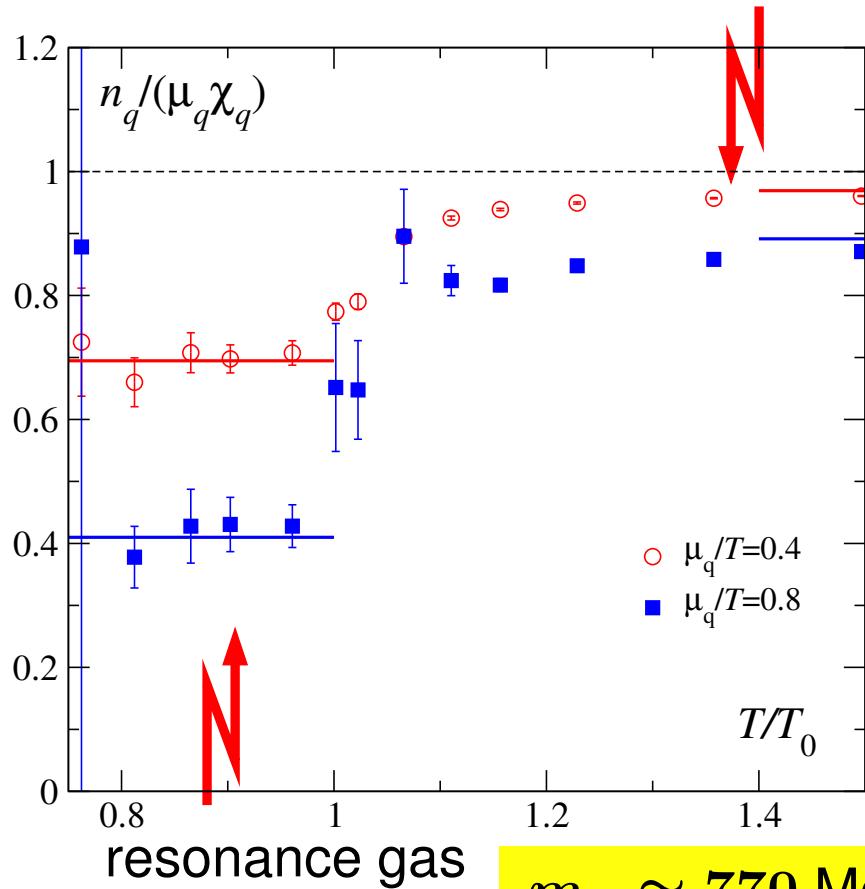
at chiral critical point

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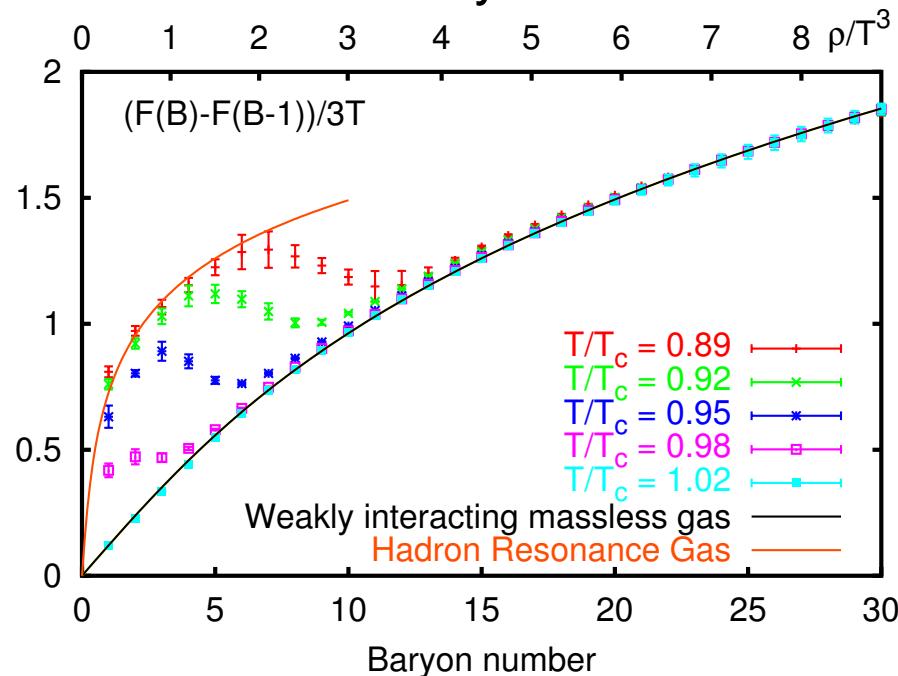
$m_\pi \simeq 770$ MeV, smaller m_q needed!!

...Status of finite density calculations

- calculations for non-vanishing chemical potential ($\mu_q > 0$)
show a rapid transition from a HRG to a QGP;
where and whether it becomes a first order transition
still is unclear

...Status of finite density calculations

- calculations for non-vanishing chemical potential ($\mu_q > 0$) show a rapid transition from a HRG to a QGP; where and whether it becomes a first order transition still is unclear
- alternative approach in the canonical ensemble ($B > 0$) looks promising; so far applied only to 4-flavor QCD where the transition always is first order



S. Kratochvila, Ph. de Forcrand,
hep-lat/0509143

In-medium modification of heavy quark bound states

- heavy quark free energies ($F = E - TS$) provide input to phenomenological modeling of finite-T potentials;
solution of Schrödinger eq. suggests dissolution of J/ψ for $T \simeq 2T_c$;
 χ_c and ψ' dissolve already at $T \simeq T_c$
- hadron correlation functions at finite-T give access to spectral functions
spectral analysis suggests that J/ψ dissolves for $T \gtrsim (1.6 - 2.0)T_c$;
 χ_c and ψ' dissolve already at $T \simeq T_c$

The origin of earlier discrepancies between both approaches seems to be understood (energy vs. free energy)

Heavy quark singlet free energy: remnants of confinement above T_c

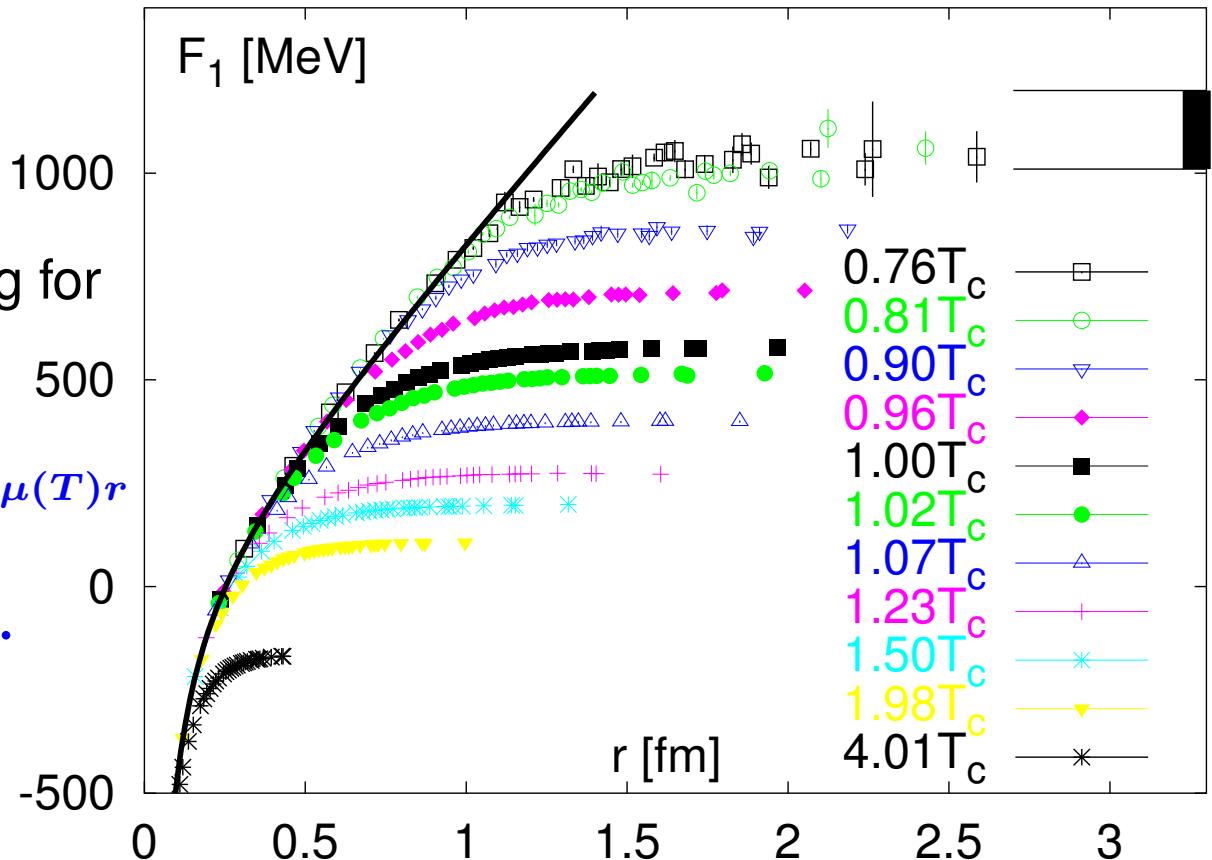
pure gauge: O.Kaczmarek, FK, P. Petreczky, F. Zantow, Phys. Rev. D70 (2004) 074505

2-flavor QCD: O.Kaczmarek, F. Zantow, Phys. Rev. D71 (2005) 114510

- singlet free energy

- $T \simeq T_c$: screening for
 $r \gtrsim 0.5$ fm

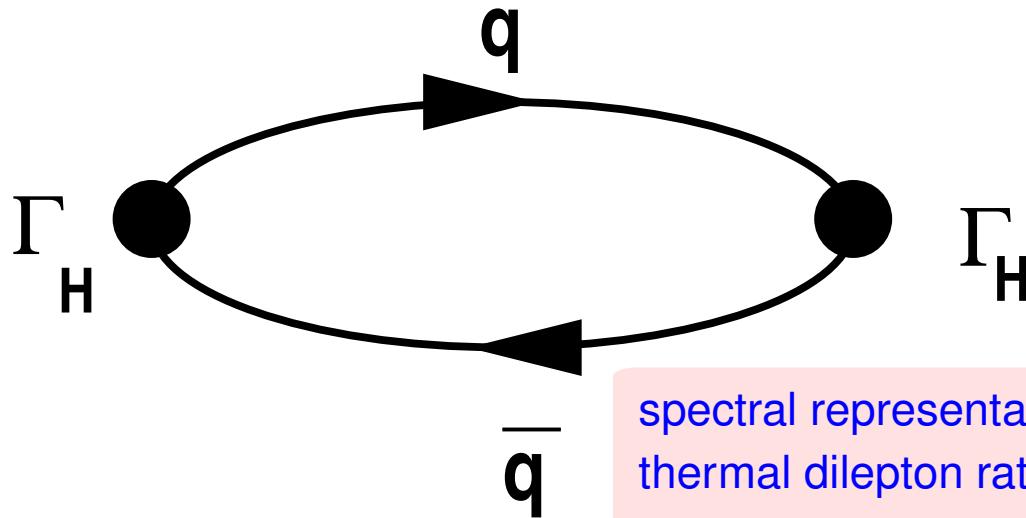
$$F_1(r, T) \sim \frac{\alpha(T)}{r} e^{-\mu(T)r} + \text{const.}$$



- $F_1(r, T)$ follows linear rise of $V_{\bar{q}q}(r, T = 0) = -\frac{4\alpha(r, T = 0)}{3r} + \sigma r$
for $T \lesssim 1.5T_c$, $r \lesssim 0.3$ fm

Thermal meson correlation functions and spectral functions

Thermal correlation functions: 2-point functions which describe propagation of a $\bar{q}q$ -pair
spectral representation of correlator \Rightarrow in-medium properties of hadrons;
thermal dilepton (photon) rates



spectral representation of
thermal dilepton rate

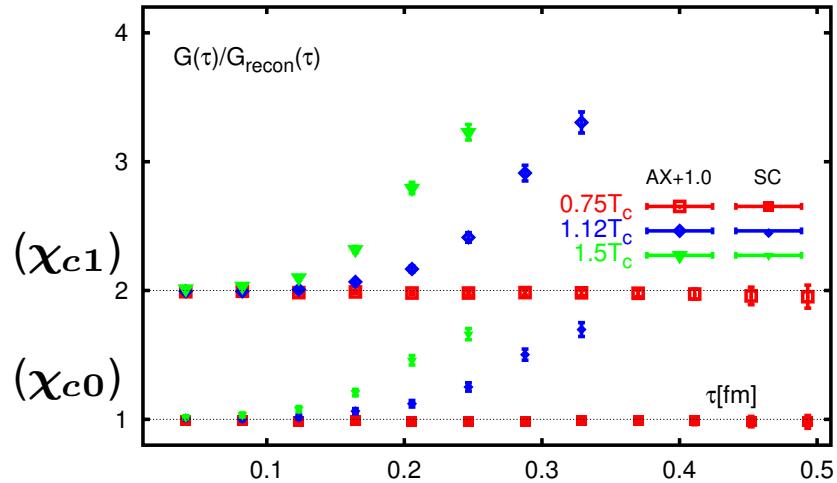
$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega, \vec{p}, T)}{\omega^2(e^{\omega/T} - 1)}$$

spectral representation of
Euclidean correlation functions

$$G_H^\beta(\tau, \vec{r}) = \int_0^\infty d\omega \int \frac{d^3\vec{p}}{(2\pi)^3} \sigma_H(\omega, \vec{p}, T) e^{i\vec{p}\cdot\vec{r}} \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

Heavy quark spectral functions and correlation functions

data for $G_H(\tau, T)$ over reconstructed correlation functions at T from data below T_c

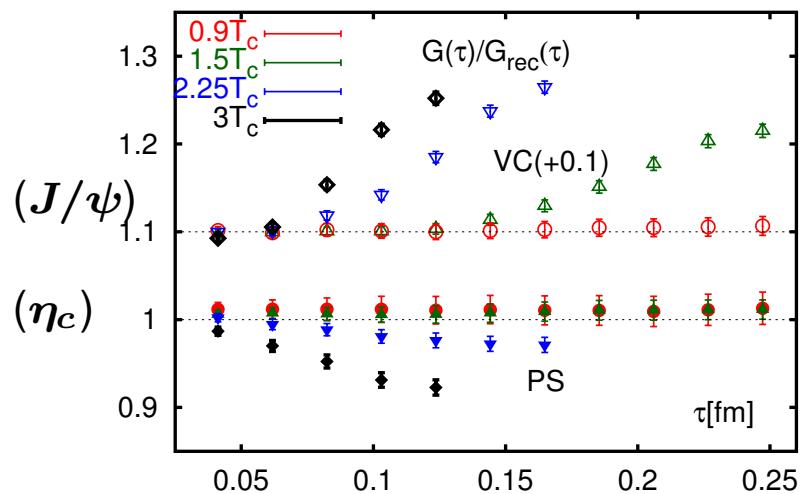


scalar and axial-vector correlation functions:

strong temperature dependence just above T_c
for χ_c states

(normalized at $T < T_c$)

($48^3 \times N_\tau$, $N_\tau = 12, 16, 24$, $a = 0.04$ fm)



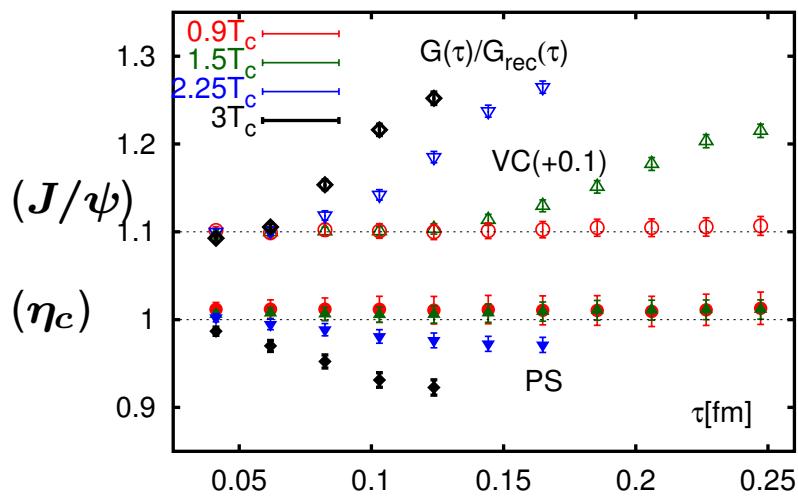
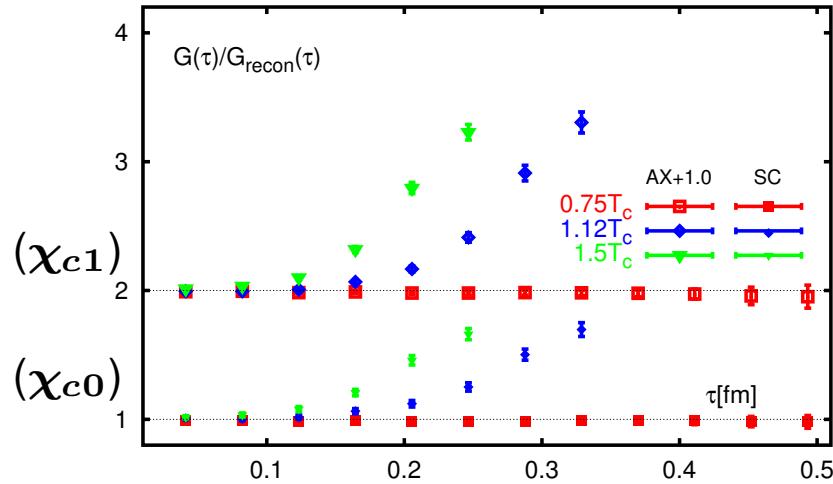
vector and pseudoscalar correlation functions:

no temperature dependence for η_c up to $1.5 T_c$;
only mild but systematic temperature dependence
of J/ψ

(normalized at $T < T_c$)

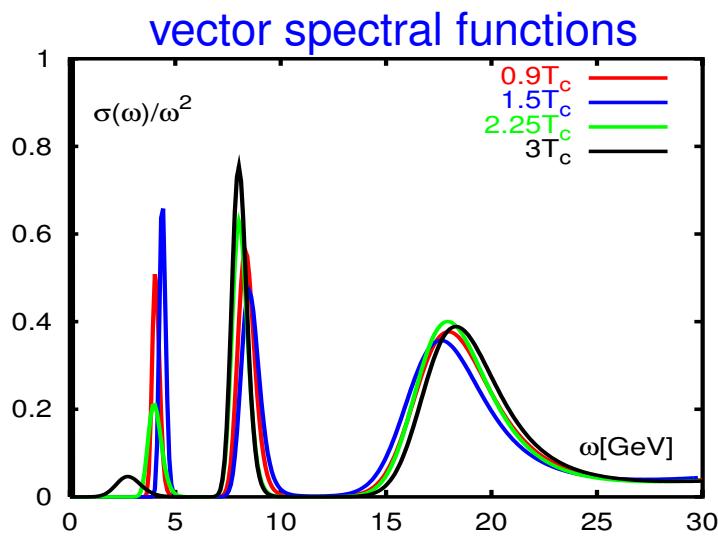
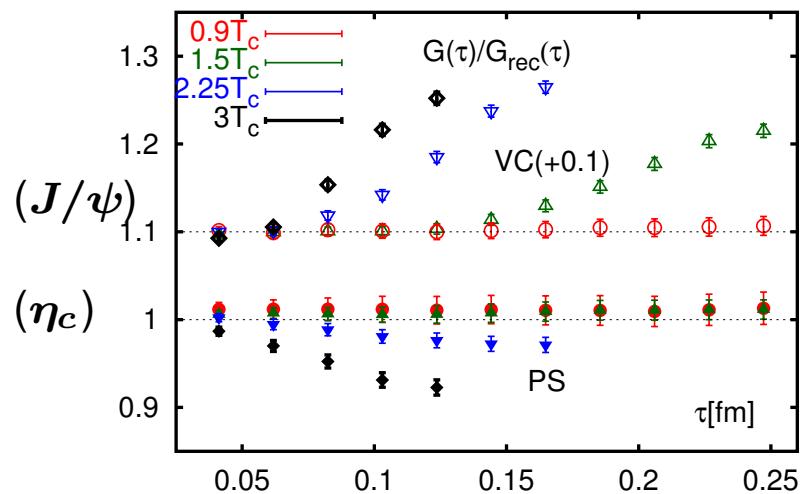
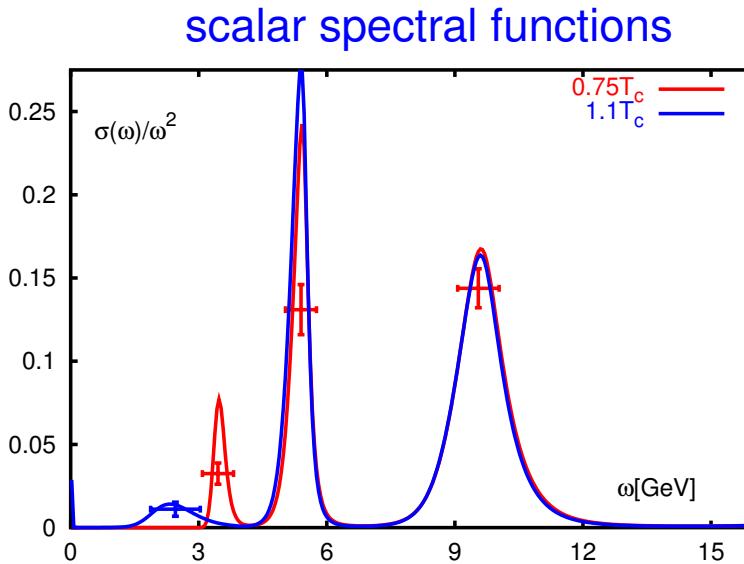
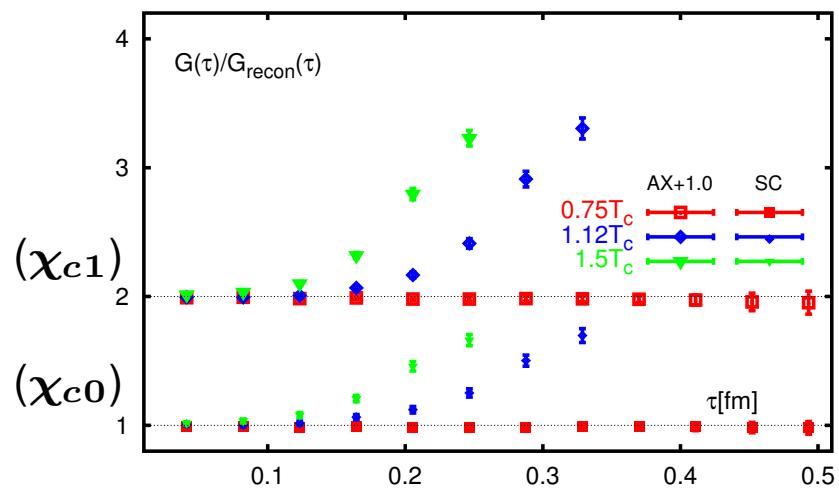
($N_\sigma = 40, 48, 64$,
 $N_\tau = 12, 16, 24, 40$, $a = 0.02$ fm)

Heavy quark spectral functions and correlation functions

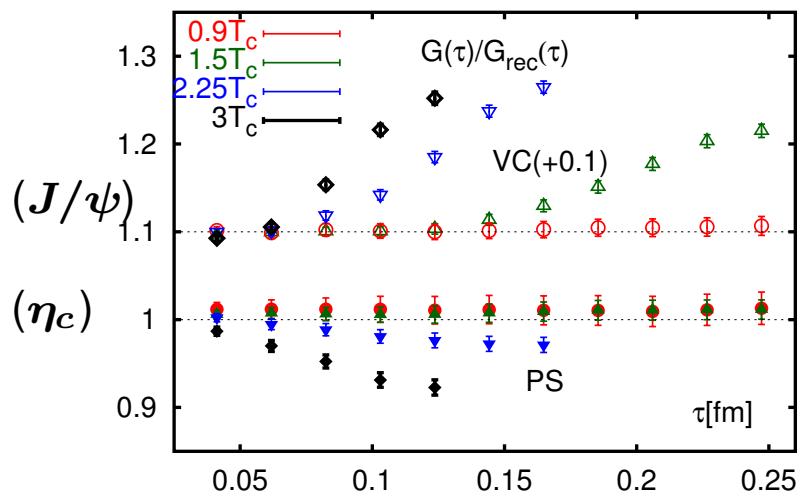
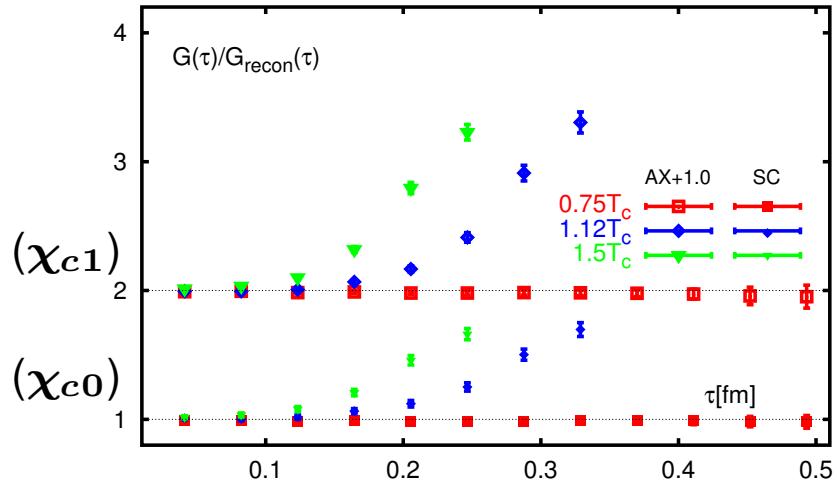


pattern seen in
correlation functions
also visible in
spectral functions

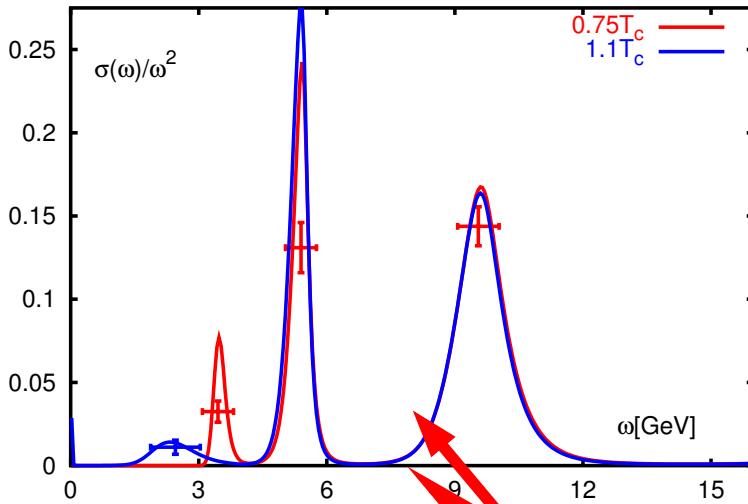
Heavy quark spectral functions and correlation functions



Heavy quark spectral functions and correlation functions



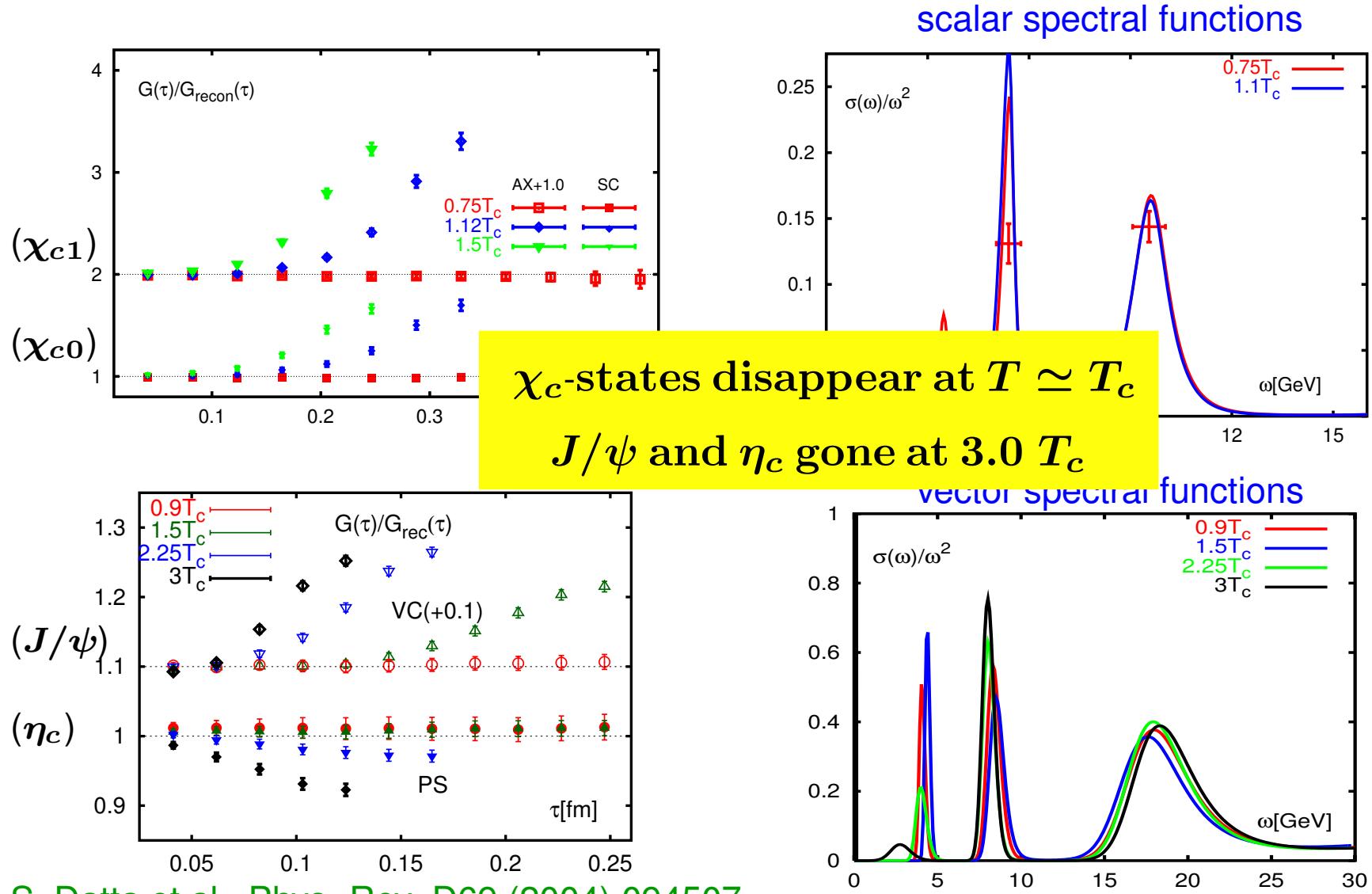
scalar spectral functions



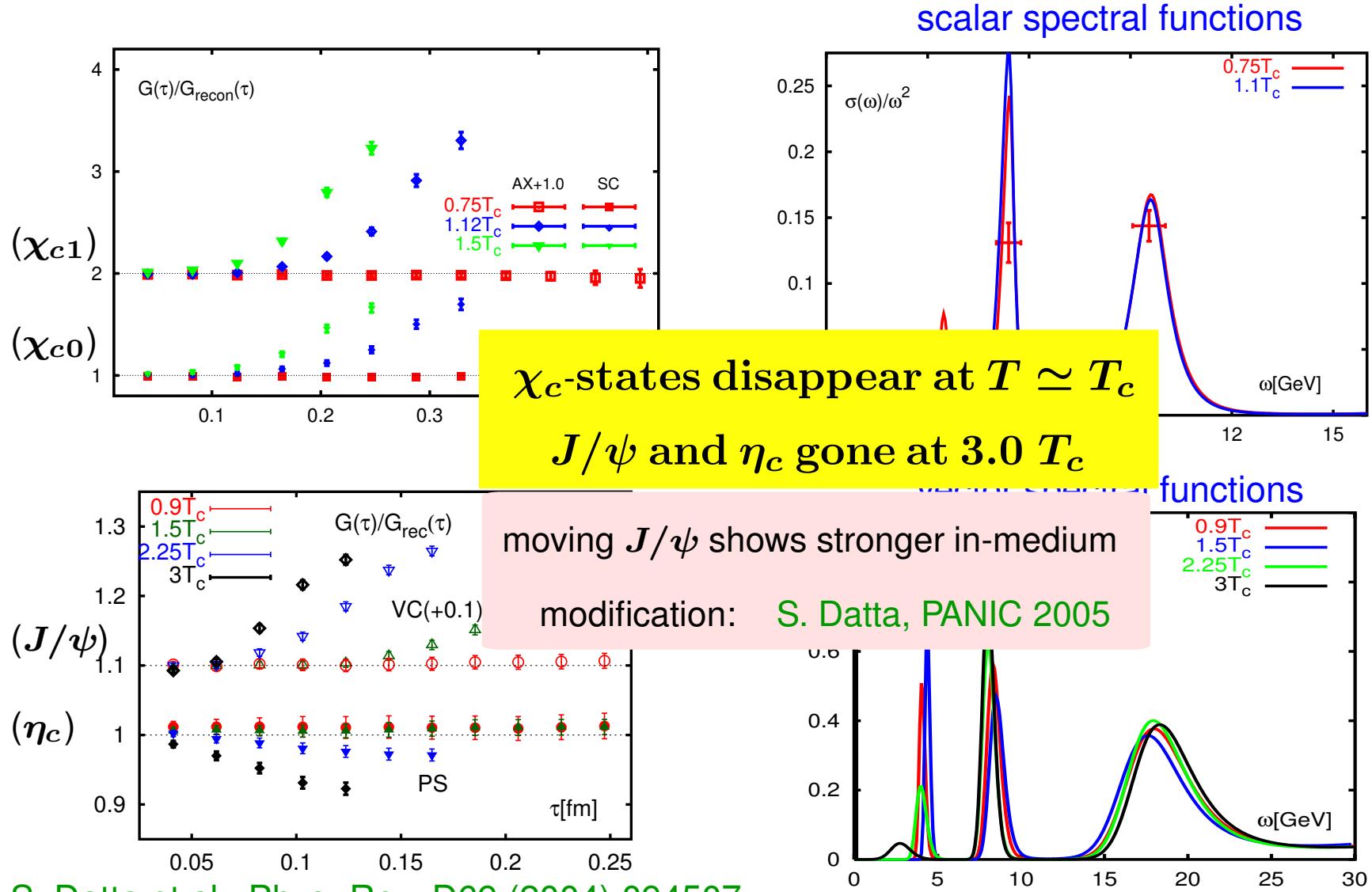
vec

ultra-violet cut-off effects;
Wilson-doubler;
finite Brillouin zone;
need to get better control
over lattice cut-off effects
resolution statistics limited

Heavy quark spectral functions and correlation functions

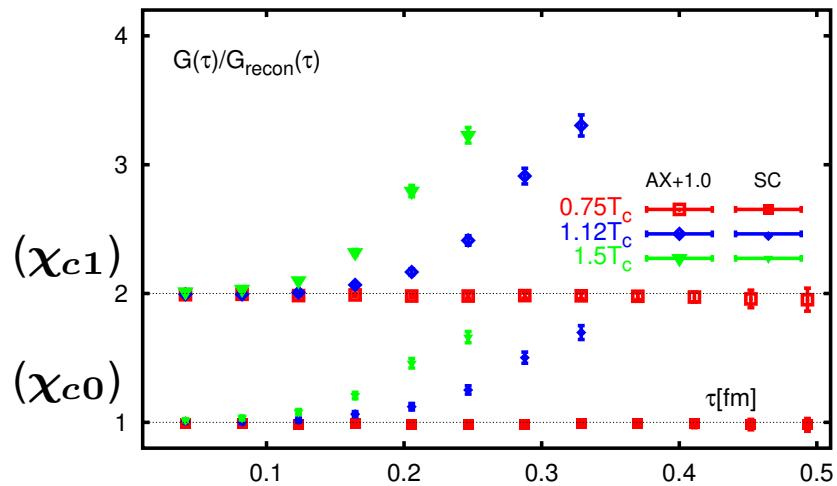


Heavy quark spectral functions and correlation functions

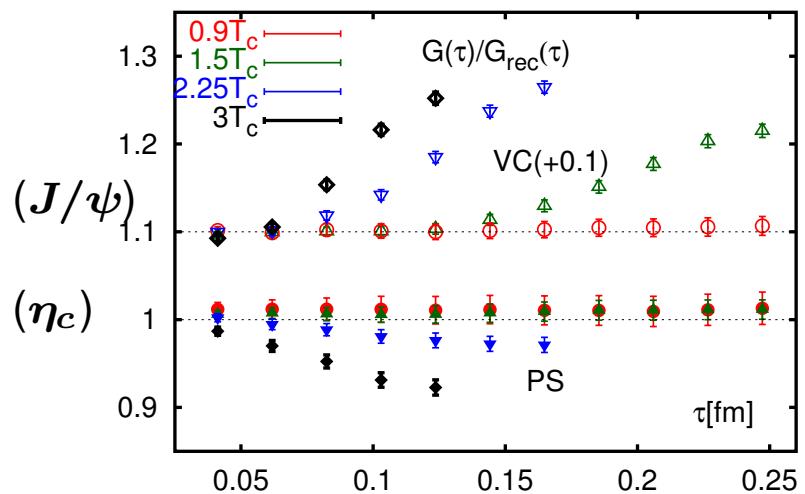
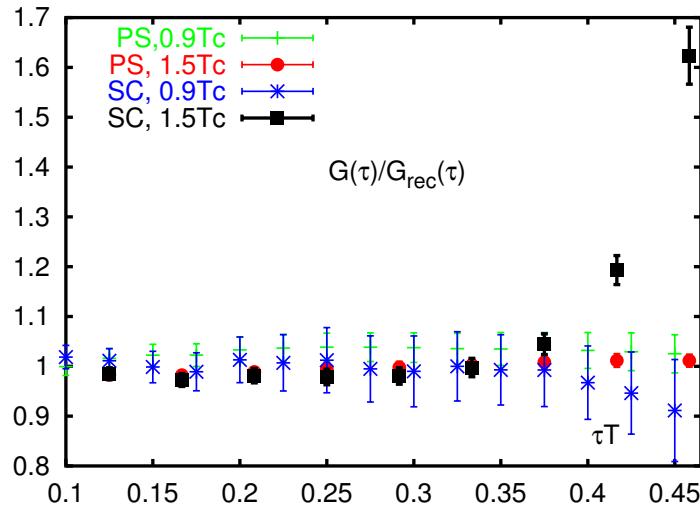


Heavy quark spectral functions and correlation functions

charm



bottom – is coming up



first results on bottomonium at high T

(more difficult, finer lattices needed)

K. Petrov et al., hep-lat/0509138

S. Datta, PANIC 2005

χ_b modified at $1.5 T_c$

η_b unmodified at $1.5 T_c$

Conclusions

- Bulk thermodynamics is currently under intensive study:
uncertainties on $T_c \simeq 175$ MeV are still about 10%;
the EoS shows little quark mass and cut-off dependence
for $T \geq T_c$.
- The last word on the QCD phase diagram is not yet spoken:
universal properties of the transition in 2-flavor QCD still have
to be established;
the location of the chiral critical point still is uncertain
- Heavy quark bound states exist well above T_c :
charmonium studies get refined, excited states dissolve
close to T_c ;
first results for bottomonium are coming up.